

Question 1. *Suppose V is a vector space over \mathbb{F} with a basis of order n . Then prove "Every basis of V has order n "*

My Answer 1. *Suppose T is another basis. Since S is independent and T is spanning, $|T| \geq |S|$. The other direction is less trivial, since T might be infinite, and Steinitz does not immediately apply. Instead, we argue as follows: since T is linearly independent, every finite subset of T is independent. Also, S is spanning. So every finite subset of T has order at most $|S|$. So $|T| \leq |S|$. So $|T| = |S|$ (copy from [Part IB - Linear Algebra](#))*